Channel Estimation using In-Band Pilots for Uplink Massive MIMO

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***Abstract*—In this paper, we extend the channel estimation technique using in-band pilots (IBP) for uplink DFT-Precoded OFDM single input single output (SISO) to massive multiple input multiple output (MIMO) system and compare its performance with that of using time domain superimposed pilots (SP). We derive analytical expression for the mean square error (MSE) of least square channel estimation using IBP and show that IBP outperforms SP when the data frequencies are removed for the accommodation of pilot frequencies. Moreover, we compare the signal-to-interference ratio (SINR), bit error rate (BER), and peak-to-average power ratio (PAPR) performance of the DFT-precoded OFDM signal with IBP and SP. It was found that the removal of a small portion of the pilot frequencies has negligible effect on the PAPR performance and that IBP gives slightly better BER and SINR performance over SP, depending on the number of antennas and the total number of users in the system.**

***Index Terms*—channel estimation, DFT-precoded OFDM, in-band pilots, superimposed pilots, uplink massive MIMO**

I. Introduction

One of the challenges of using conventional time multiplexed pilots to estimate the channel state information (CSI) in the uplink massive multiple input multiple output (MIMO) is the problem of pilot contamination when obtaining the CSI at the base station (BS) [1]–[3]. The same frequency bands are used in a multi-cell communication system. If the same pilot signals are shared by different cells, the pilot signals received by the BS in a particular cell can be contaminated by the same pilot signal coming from the cells other than this particular cell. Orthogonal pilots can be used to mitigate the problem of pilot contamination in massive MIMO system [4], [5]. However, when the coherence interval is short, i.e. communication channel is changing at a fast rate, the overhead due to orthogonal time multiplexed pilots can be significant such that it makes a massive MIMO system inefficient [6].

One solution to the short coherence interval issue of using conventional time multiplexed pilots is to use time domain superimposed pilots for channel estimation [7]–[9]. By superimposing the pilot signal on the data signal in the time domain, one can replace the time needed for pilot with data transmission at the expense of suffering the interference from the data signals from all users in the system. This degrades the initial performance of the channel estimation, resulting in performance degradation during the data detection phase. Although iterative data decision-directed interference cancellation technique can be used to mitigate the interference after the initial channel estimation [7]–[10], it increases the receiver processing complexity.

The key to improve the initial performance of the channel estimation is to remove the interference from the data signal, which is not possible when the pilot signals are superimposed in the time domain. However, it is possible to completely remove the interference from the data signal when the pilots are superimposed in the frequency domain, called in-band pilots (IBP), at the expense of slightly distorting the original data signal. IBP refers to pilots being superimposed in the frequency domain on the data frequencies [11], [12]. One of the advantages of using in-band pilots is the flexibility of adjusting the proportion of the power between pilot and data frequencies such that controlled interference from the data symbols can be adjusted based on different application scenarios. When data frequencies are completely removed for the accommodation of pilot frequencies, there is no data interference at the pilot locations from the reference user itself. Moreover, in-band pilots allow processing of single carrier (SC) systems with frequency-domain equalization, which has the advantage of lower peak-to-average power requirement for the amplifier [13]–[15]. With the generalized multi-carrier (GMC) [16], [17], a SC signal can be generated by Discrete Fourier Transform (DFT)-precoding an OFDM signal [12]. Moreover, generating SC signals in the frequency domain avoids conventional complex symbol-by-symbol filtering and affords an extra degree of spectrum flexibility, which are useful for both single or multiple user scenarios.

In this paper, we extend the channel estimation technique using in-band pilots (IBP) for uplink single input and single output DFT-precoded OFDM system proposed in [11] to massive MIMO systems and compare its performance with that of using conventional time domain superimposed pilots. We derive analytical expression for the mean square error (MSE) of the channel estimation using IBP. Moreover, we compare the signal-to-interference ratio (SINR) and bit error rate (BER) performance of the signal with IBP and with superimposed pilots in the time domain (SP). In addition, we also evaluate the effect on the peak-to-average power ratio (PAPR) of the distorted signal when data frequencies are removed or scaled for superimposing the IBP pilots.

The rest of the paper is organized as follows: Sec. II

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describes the system model, followed by the description of generating DFT-precoded OFDM signals with IBP in Sec.III. In Sec. IV, least square (LS) channel estimation with IBP, along with the derivation of MSE, is provided. Data detection with channel estimation using IBP is described in Sec. V. Finally, simulation results are presented in Sec. VI and the conclusions in Sec. VII.

II. System Model

We consider an uplink DFT-Precoded OFDM massive MIMO system with L cells and K user terminals per cell. Each cell contains a BS equipped with $M \gg K$ antennas. We assume that the channel is constant for C_u symbols during the uplink transmission. Hence, the time domain signal received by the M antennas at the jth BS, denoted by $\mathbf{Y}_j \in \mathbb{C}^{M\times C_u}$ is given as [8],

$$\mathbf{Y}_j = \sum_{l=0}^{L-1}\sum_{k=0}^{K-1} \sqrt{\mu_{l,k}}\mathbf{h}_{j,l,k}\mathbf{s}_{l,k}^T + \mathbf{W}_j \tag{1}$$

where $\mu_{l,k}$ denotes the transmit power for the kth user in the lth cell and $\mathbf{h}_{j,l,k} \in \mathbb{C}^{M\times 1}$ contains the CSI between the kth user equipment in the lth cell to the M antennas of the jth BS, $\mathbf{s}_{l,k} \in \mathbb{C}^{C_u\times 1}$, is the data symbols transmitted from the kth user in the lth cell, and $\mathbf{W}_j \in \mathbb{C}^{M\times C_u}$ is the additive white Gaussian noise at the jth BS with each column distributed as $CN(\mathbf{0}, \sigma^2\mathbf{I_M})$ and mutually independent of the other columns. We assume $\mathbf{h}_{j,l,k}$ is distributed as [8],

$$\mathbf{h}_{j,l,k} \sim CN(\mathbf{0}, \beta_{j,l,k}\mathbf{I_M}) \tag{2}$$

where $\beta_{j,l,k}$ denotes the large-scale path-loss coefficient which depends on the user location in the cell and is assumed to be known at the BS. We further assume that all elements in $\mathbf{h}_{j,l,k}$ are mutually independent of each other and that $\mathbf{h}_{j,l,k}$ is constant during the transmission of C symbols, which includes uplink pilots, data and downlink data.

III. Superimposing Pilots in the Frequency Domain

Consider a DFT-precoded OFDM modulated uplink transmission scheme [12]. By taking the DFT of (1) using the Fourier Transform matrix $\mathbf{F} \in \mathbb{C}^{C_u\times C_u}$, given as

$$\mathbf{F} = \frac{1}{\sqrt{C_u}}\begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & e^{j\frac{2\pi}{C_u}} & \cdots & e^{j\frac{2\pi(C_u-1)}{C_u}} \\ 1 & e^{j\frac{4\pi}{C_u}} & \cdots & e^{j\frac{4\pi(C_u-1)}{C_u}} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & e^{j\frac{2\pi(C_u-1)}{C_u}} & \cdots & e^{j\frac{2\pi(C_u-1)^2}{C_u}} \end{bmatrix} \tag{3}$$

, we have the received signal $\mathbf{Y}_j$ in the frequency domain $\mathbb{Y}_j \in \mathbb{C}^{M\times C_u}$, given as

$$\begin{aligned}\mathbb{Y}_j &= \mathbf{Y}_j\mathbf{F} \\ &= \left(\sum_{l=0}^{L-1}\sum_{k=0}^{K-1} \sqrt{\mu_{l,k}}\mathbf{h}_{j,l,k}\mathbf{s}_{l,k}^T\right)\mathbf{F} + \mathbf{W}_j\mathbf{F} \\ &= \sum_{l=0}^{L-1}\sum_{k=0}^{K-1} \sqrt{\mu_{l,k}}\mathbf{h}_{j,l,k}\mathbb{S}_{l,k}^T + \mathbb{W}_j\end{aligned} \tag{4}$$

where $\mathbb{S}_{l,k}^T = \mathbf{s}_{l,k}^T\mathbf{F} \in \mathbb{C}^{1\times C_u}$ are the transmitted signal (data, pilots, or both) in the frequency domain and $\mathbb{W}_j \in \mathbb{C}^{M\times C_u}$ are the noise samples in the frequency domain.

Assume that the channel response is constant during the coherence interval and that a unique orthogonal pilot can be assigned for each of the K users in L cells, i.e. $KL \leq C_u$. For the kth user in the lth cell, the pilots can be superimposed in the frequency domain onto its data frequencies as follows,

$$\mathbb{S}_{l,k} = \mathbf{Q}_{l,k}\mathbb{X}_{l,k} + \lambda_{l,k}\mathbb{P}_{l,k} \tag{5}$$

where $\mathbb{X}_{l,k}$ and $\mathbb{P}_{l,k}$ represents the data and pilot frequencies, respectively, for the kth user in the lth cell, $\lambda_{l,k}$ is the scaling factor for the power of the (l,k)th pilot frequency $\mathbb{P}$, and $\mathbf{Q}_{l,k} \in \mathbb{R}^{C_u\times C_u}$ is a location matrix for the allocation of pilots for the kth user in the lth cell. $\mathbf{Q}_{l,k}$ can be defined as a square diagonal matrix with the following main diagonal elements,

$$\mathbf{q}_{l,k} = \begin{bmatrix} \rho_d & \cdots & \rho_d & \rho_{l,k} & \rho_d & \cdots & \rho_d \end{bmatrix} \tag{6}$$

where $\rho_{l,k}$ is the scaling factor for the power of the (l,k)th data frequency where the pilot frequency is superimposed and ρ_d is the scaling factor for the remaining data frequencies. We assume that all the data frequencies at the non-pilot locations are scaled by the same scaling factor ρ_d and that

$$\frac{C_u-1}{C_u}\rho_d^2 + \frac{1}{C_u}\rho_{l,k}^2 + \lambda_{l,k}^2 = 1. \tag{7}$$

When $\rho_{l,k}$ is set to 0, the data frequencies are completely removed, resulting in no data interference from the kth user in the lth cell. When the data frequency at the pilot location has the same scaling factor as the other data frequencies at the non-pilot locations, i.e. $\rho_{l,k} = \rho_d$, the data signal will not be distorted. However, the uneven scaling of data frequencies, i.e. $\rho_{l,k} \neq \rho_d$, for superimposing pilot frequencies in the signal bandwidth affects the PAPR and BER of the DFT-precoded OFDM signal. The degradation level depends on the ratio of pilot and data frequencies in a data block $\eta \triangleq n_p/n_b$, where n_p is the number of pilot frequencies to be scaled and n_b is the block length including both data and pilot frequencies [11]. Larger η value results in larger PARP and BER performance degradation. In this paper, we have $\eta = 1/C_u$ since only one frequency is needed for superimposing the IBP pilot in each uplink block of data C_u.

By substituting (5) into (4), the frequency domain received signal with IBP can be written as,

$$\mathbb{Y}_j = \sum_{l=0}^{L-1}\sum_{k=0}^{K-1} \sqrt{\mu_{l,k}}\mathbf{h}_{j,l,k}\left(\mathbf{Q}_{l,k}\mathbb{X}_{l,k} + \lambda_{l,k}\mathbb{P}_{l,k}\right)^T + \mathbb{W}_j. \tag{8}$$

where $\mathbb{X}_{l,k} \in \mathbb{C}^{C_u\times 1}$ and $\mathbb{P}_{l,k} \in \mathbb{C}^{C_u\times 1}$. We assume the pilots are orthogonal in the frequency domain, i.e.

$$\mathbb{P}_{l,k}^H\mathbb{P}_{n,p} = C_u\delta_{l,n}\delta_{k,p} \tag{9}$$

where $\mathbb{P}_{l,k}^T = \begin{bmatrix} 0 & 0 & \cdots & P_{l,k} & \cdots & 0 \end{bmatrix}$ and $P_{l,k} \in \mathbb{C}$ with a magnitude of $\sqrt{C_u}$. Note that the length of the pilot is the same as that of data symbols C_u.

IV. Channel Estimation using In-Band Pilots

The least square (LS) estimate of the channel for the mth user in jth cell at the jth BS can be written as,

$$\begin{aligned}\hat{\mathbf{h}}^{\text{IBP}}_{j,j,m} &\triangleq \frac{1}{C_u\sqrt{\mu_{j,m}}\lambda_{j,m}}\mathbb{Y}_j\mathbb{P}^*_{j,m}\\ &= \frac{1}{C_u\sqrt{\mu_{j,m}}\lambda_{j,m}}\left(\sum_{l=0}^{L-1}\sum_{k=0}^{K-1}\sqrt{\mu_{l,k}}\mathbf{h}_{j,l,k}\lambda_{l,k}\mathbb{P}^T_{l,k}\mathbb{P}^*_{j,m}\right.\\ &\left.+\sum_{l=0}^{L-1}\sum_{k=0}^{K-1}\sqrt{\mu_{l,k}}\mathbf{h}_{j,l,k}(\mathbf{Q}_{l,k}\mathbb{X}_{l,k})^T\mathbb{P}^*_{j,m}+\mathbb{W}_j\mathbb{P}^*_{j,m}\right)\\ &= \mathbf{h}_{j,j,m}+\frac{\rho_{j,m}}{C_u\lambda_{j,m}}\chi^{(j,m)}_{j,m}P^{(j,m)*}_{j,m}\mathbf{h}_{j,j,m}\\ &+\frac{\rho_d P^{(j,m)*}_{j,m}}{C_u\lambda_{j,m}\sqrt{\mu_{j,m}}}\sum_{\substack{l=0\\ \{l\neq j,k\neq m\}}}^{L-1}\sum_{k=0}^{K-1}\sqrt{\mu_{l,k}}\chi^{(j,m)}_{l,k}\mathbf{h}_{j,l,k}\\ &+\frac{\mathbb{W}_j\mathbb{P}^*_{j,m}}{C_u\lambda_{j,m}\sqrt{\mu_{j,m}}}\end{aligned} \tag{10}$$

where $\chi^{(j,m)}_{l,k}$ denotes the frequency response at the $(j\times K+m)$th location of $\mathbb{X}_{l,k}\in\mathbb{C}^{C_u\times 1}$ for the kth user in the lth cell. $P^{(j,m)}_{j,m}$ denotes the pilot frequencies for the $(j\times K+m)$th location of $\mathbb{P}_{j,m}$ for the mth user in the jth cell. From the 2nd and 3rd term of (10), we can see that the channel is contaminated at the $(j\times K+m)$th frequency from all users, if $\rho_{j,m}\neq 0$. Assume that all the users have unit transmit power, i.e. $\sqrt{\mu_{l,k}}=1,\forall l,k$ and $\rho_{j,m}=0$. The contamination from the data frequency for the mth user in the jth cell can be removed and (10) can then be simplified as

$$\begin{aligned}\tilde{\mathbf{h}}^{\text{IBP}}_{j,j,m} &\triangleq \mathbf{h}_{j,j,m}+\frac{\rho_d}{C_u\lambda_{j,m}}\sum_{\substack{l=0\\ \{l\neq j,k\neq m\}}}^{L-1}\sum_{k=0}^{K-1}\chi^{(j,m)}_{l,k}P^{(j,m)*}_{j,m}\mathbf{h}_{j,l,k}\\ &+\frac{1}{C_u\lambda_{j,m}}\mathbb{W}_j\mathbb{P}^*_{j,m}\end{aligned} \tag{11}$$

From (11), the contamination comes from the data frequencies at the $(j\times K+m)$th location for all users, except for the mth user in the jth cell.

Let's compare the corresponding estimated channel using superimposed pilots in the time domain, given as [8]

$$\begin{aligned}\tilde{\mathbf{h}}^{\text{SP}}_{j,j,m} &\triangleq \mathbf{h}_{j,j,m}+\frac{\rho_d}{C_u\lambda_{j,m}}\sum_{l=0}^{L-1}\sum_{k=0}^{K-1}\mathbf{h}_{j,l,k}\mathbf{x}^T_{l,k}\mathbf{p}^*_{j,m}\\ &+\frac{1}{C_u\lambda_{j,m}}\mathbf{W}_j\mathbf{p}^*_{j,m}\end{aligned} \tag{12}$$

where $\mathbf{p}_{j,m}\in\mathbb{C}^{C_u\times 1}$ is the orthogonal pilot sequence in the time domain. From (11), there is no inference from the reference user itself. However, from (12) we can see that for time domain superimposed pilots, there still exists interference from the reference user when there is no interference from the other users, i.e. when $L=1$ and $K=1$.

Next, let's find the MSE of the LS channel estimation using IBP. Assuming that the zero-mean and mutually independent channels vectors are asymptotically orthogonal when the number of antennas M grows without bound, we have [2], [8],

$$\lim_{M\to\infty}\frac{\mathbf{h}^H_{j,l,k}\mathbf{h}_{p,q,r}}{M}=\beta_{j,l,k}\delta_{j,p}\delta_{l,q}\delta_{k,r},\forall j,l,k,p,q,r \tag{13}$$

where $\beta_{j,l,k}$ denotes the large-scale path-loss coefficient for the kth user in the j cell. Using (10) and assuming that M is large, the MSE of the channel estimation using IBP can be calculated as,

$$\begin{aligned}\text{MSE}^{\text{IBP}}_{j,j,m} &\triangleq \frac{1}{M}\mathbb{E}\left\{\left\|\hat{\mathbf{h}}_{j,j,m}-\mathbf{h}_{j,j,m}\right\|^2\right\}\\ &= \frac{\rho^2_{j,m}}{\lambda^2_{j,m}C_u}\beta_{j,j,m}+\frac{\rho^2_d}{\lambda^2_{j,m}C_u}\sum_{\substack{l=0\\ \{l\neq j,k\neq m\}}}^{L-1}\sum_{k=0}^{K-1}\beta_{j,l,k}\\ &+\frac{\sigma^2}{\lambda^2_{j,m}C_u}\end{aligned} \tag{14}$$

From (14), the channel estimation errors result from the data frequencies at the pilot locations from the interfering users and the noise. The first term in (14) is the interference due to the data frequency that is superimposed with the pilot frequency for the reference user.

To remove the effect of the data tone on the pilot tone for the mth user in the jth cell, we can set $\rho_{j,m}=0$. Note that the self-interference due to data symbols for channel estimation cannot be removed when the pilots are superimposed in the time domain. Thus, (14) can be further simplified into,

$$\text{MSE}^{\text{IBP0}}_{j,j,m}=\frac{\rho^2_d}{\lambda^2_{j,m}C_u}\sum_{\substack{l=0\\ \{l\neq j,k\neq m\}}}^{L-1}\sum_{k=0}^{K-1}\beta_{j,l,k}+\frac{\sigma^2}{\lambda^2_{j,m}Cu} \tag{15}$$

where ρ_d denotes the scaling factor for the data frequencies at the non-pilot locations, while C_u is the length of the pilot sequence. Let's compare (15) with the MSE using the time domain superimposed pilots with the same parameters, which are given as [10],

$$\text{MSE}^{\text{SP}}_{j,j,m}=\frac{\rho^2_d}{\lambda^2_{j,m}C_u}\left(\sum_{k=0}^{K-1}\beta_{j,j,k}+\sum_{\substack{l=0\\ l\neq j}}^{L-1}\sum_{k=0}^{K-1}\beta_{j,l,k}\right)+\frac{\sigma^2}{\lambda^2_{j,m}Cu} \tag{16}$$

The difference between (16) and (15) can be calculated as

$$\begin{aligned}\Delta\text{MSE}^{\text{SP-IBP0}}_{j,j,m} &\triangleq \text{MSE}^{\text{SP}}_{j,j,m}-\text{MSE}^{\text{IBP0}}_{j,j,m}\\ &= \frac{\rho^2_d}{\lambda^2_{j,m}C_u}\beta_{j,j,m}\end{aligned} \tag{17}$$

From (17), it is obvious that the MSE difference for using SP and IBP depends on the length of pilot sequence C_u and the large-scale path-loss coefficient for the mth user in the jth cell $\beta_{j,j,m}$. The difference diminishes for large value of C_u.

In general, using (16) and (14), the MSE difference can be calculated as,

$$\begin{aligned}\Delta\text{MSE}^{\text{SP-IBP}}_{j,j,m} &\triangleq \text{MSE}^{\text{SP}}_{j,j,m} - \text{MSE}^{\text{IBP}}_{j,j,m} \\ &= \frac{\rho_d^2 - \rho_{j,m}^2}{\lambda_{j,m}^2 C_u}\beta_{j,j,m}.\end{aligned} \tag{18}$$

When $\rho_d^2 = \rho_{j,m}^2$, since all data frequencies, including pilot frequencies, are scaled by the same factor ρ_d, the data signal is not distorted by the superimposing of frequency domain pilot. As a result, the MSE performance for superimposing the pilots in the frequency domain is the same as superimposing the pilots in the time domain.

V. Data Detection with In-band Pilots

To estimate the data frequencies, the scaled pilot frequencies are removed first. Given the estimated channel $\hat{\mathbf{h}}^{\text{IBP}}_{j,j,m}$ and using (8), the received signal with the removal the pilot frequencies can be found as,

$$\mathbb{Y}_j' = \mathbb{Y}_j - \sqrt{\mu_{j,m}}\lambda_{j,m}\hat{\mathbf{h}}^{\text{IBP}}_{j,j,m}\mathbb{P}^T_{j,m} \tag{19}$$

where $\lambda_{j,m}$ is the scaling factor for the pilot frequencies for the mth user in the jth cell. In this paper, we assume that all users allocate the same proportion of pilot and data power. From (10), $\hat{\mathbf{h}}_{j,j,m}$ can be written as,

$$\hat{\mathbf{h}}_{j,j,m} = \mathbf{h}^{\text{IBP}}_{j,j,m} + \Delta\mathbf{h}_{j,j,m} \tag{20}$$

where $\Delta\mathbf{h}_{j,j,m}$ denotes the channel estimation errors. Let's assume matched filter (MF) detection. The scaled received data frequencies, denoted as $\hat{\mathbb{X}}_j$, can be recovered by MF detection, given as

$$\hat{\mathbb{X}}^T_{j,j,m} = \frac{1}{M}\hat{\mathbf{h}}^H_{j,j,m}\mathbb{Y}_j' = \mathbf{S}_{j,j,m} + \mathbf{I}_{j,j,m} \tag{21}$$

where $\mathbf{S}_{j,j,m}$ denotes the estimated data frequencies for the mth user in the jth cell, given as

$$\mathbf{S}_{j,j,m} \triangleq \frac{1}{M}\sum_{l=0}^{L-1}\sum_{k=0}^{K-1}\mathbf{h}^H_{j,j,m}\mathbf{h}_{j,l,k}\left(\mathbf{Q}_{l,k}\mathbb{X}_{l,k}\right)^T \tag{22}$$

and $\mathbf{I}_{j,j,m}$ denotes the total interference for the mth user in the jth cell, given as

$$\begin{aligned}\mathbf{I}_{j,j,m} \triangleq\ & \frac{1}{M}\sum_{l=0}^{L-1}\sum_{k=0}^{K-1}\Delta\mathbf{h}^H_{j,j,m}\mathbf{h}_{j,l,k}\left(\mathbf{Q}_{l,k}\mathbb{X}_{l,k}\right)^T \\ &+ \frac{1}{M}\sum_{\substack{l=0\\ l\neq j}}^{L-1}\sum_{\substack{k=0\\ k\neq m}}^{K-1}\lambda_{l,k}\mathbf{h}^H_{j,j,m}\mathbf{h}_{j,l,k}\mathbb{P}^T_{l,k} \\ &+ \frac{1}{M}\sum_{\substack{l=0\\ \{l\neq j,}}^{L-1}\sum_{\substack{k=0\\ k\neq m\}}}^{K-1}\lambda_{l,k}\Delta\mathbf{h}^H_{j,j,m}\mathbf{h}_{j,l,k}\mathbb{P}^T_{l,k} \\ &- \frac{1}{M}\lambda_{j,m}\mathbf{h}^H_{j,j,m}\Delta\mathbf{h}_{j,j,m}\mathbb{P}^T_{j,m} \\ &- \frac{1}{M}\lambda_{j,m}\left\|\Delta\mathbf{h}_{j,j,m}\right\|^2\mathbb{P}^T_{j,m} \\ &+ \frac{1}{M}\mathbf{h}^H_{j,j,m}\mathbb{W}_j + \frac{1}{M}\Delta\mathbf{h}^H_{j,j,m}\mathbb{W}_j.\end{aligned} \tag{23}$$

From (22) and (23), the SINR can be calculated as

$$\text{SINR}^{\text{IBP}}_{j,j,m} \triangleq \frac{\mathbb{E}\{\|\mathbf{S}_{j,j,m}\|^2\}}{\mathbb{E}\{\|\mathbf{I}_{j,j,m}\|^2\}} \tag{24}$$

By taking the inverse DFT of (21), the estimated data symbols and decision operation can be written as

$$\tilde{\mathbf{x}}^T = \hat{\mathbb{X}}^T_{j,j,m}\mathbf{F}^H \tag{25}$$

$$\hat{\mathbf{x}}^T = \zeta(\tilde{\mathbf{x}}^T) \tag{26}$$

where $\mathbf{F}^H$ denotes the inverse DFT operation and $\zeta(\cdot)$ denotes data decision operation.

VI. Simulation Results

In most cases of the following simulation results to validate our derivations, we assume $L = 7$, $K = 5$, $C_u = 128$, $\rho_{j,m} = 0$, QPSK modulation, and SNR $= 10$ dB. For simplicity, we set large scale coefficient $\beta_{j,j,k} = 1, \forall k$ in the center cell and the corresponding coefficients for the neighboring cells as $\beta_{j,l,k} = 0.1, \forall l, k$, except for $l \neq j$ as in [18]. We compare the complementary cumulative distribution function (CCDF) of the PAPR of DFT-precoded OFDM signals with IBP and OFDM signals. We also compare the MSE performance of LS channel estimation with IBP and SP. Moreover, we evaluate and compare the SINR and BER performance with channel estimation using IBP and SP with different values of M a nd total number users $L \times K$ in the system.

A. CCDF of DFT-Precoded OFDM Signal with IBP

Fig. 1 shows the CCDF of PAPR of DFT-Precoded OFDM with different uplink data block length C_u and an upsampling factor of 4. Note that SC signal means DFT-precoded OFDM signal. The effect on PAPR for removing one pilot frequency is not significant, comparing with the DFT-precoded OFDM signals without pilots. At CCDF = 10^{-4}, the SC signal with IBP has about 0.5 dB advantage over SC signal without pilots and about 3dB advantage over OFDM signals with no pilots. It was found that the removal of a very small portion of the pilot frequencies (e.g. $\eta = 1/C_u \leq 1.56\%$) has a negligible effect on the degradation of the PAPR performance.

B. MSE Comparison

Fig. 2 depicts the validation of analytical evaluation of the MSE for the LS channel estimation using IBP and the MSE comparison between IBP and SP with different values of C_u, $M = 256$ and $\rho_{l,k} = 0, \forall l, k$. The theoretical results are consistent with the simulation results. As expected, the MSE for both IBP and SP drops significantly with the increase value of C_u. Moreover, IBP outperforms SP for small values of C_u and $K \times L$ is small. Fig. 3 shows the difference in MSE for channel estimation using IBP and SP with different proportion of the data power at the pilot location $\rho^2_{j,m}$ and pilot power $\lambda^2_{j,m}$. When $\rho_d = \rho_{j,m}$, the MSE difference between IBP and SP is zero, while smaller data power results in better initial channel estimation performance for SP.

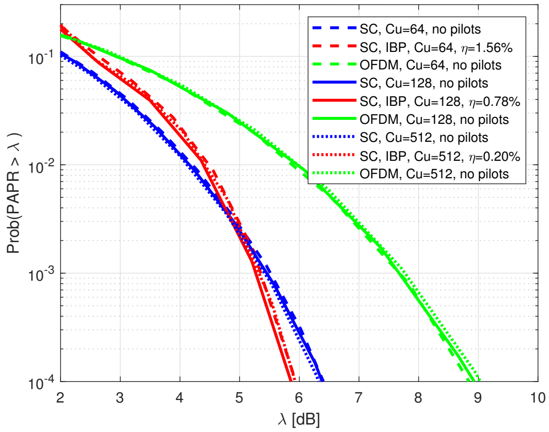

Fig. 1. CCDF of PAPR of OFDM and SC Signals with SP and IBP Pilots

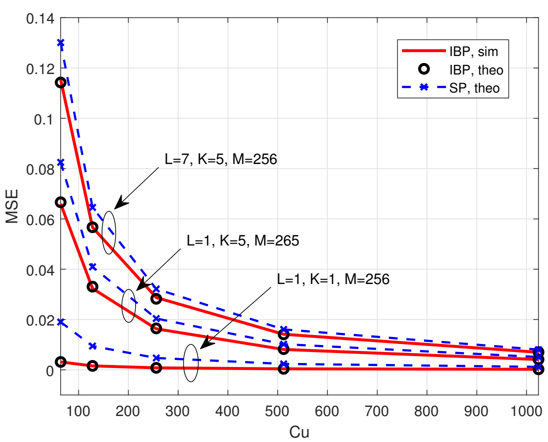

Fig. 2. MSE of Channel Estimation using IBP and SP, SNR $= 10$dB

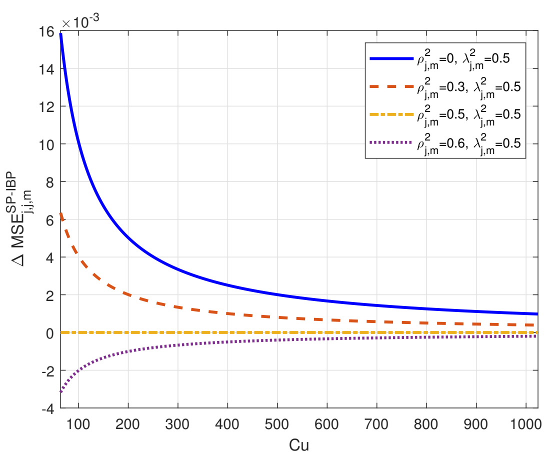

Fig. 3. Difference of MSE for IBP and SP, $\beta_{j,j,m} = 1$

C. SINR and BER Comparison

Fig. 4 shows the SINR comparison between LS channel estimation using IBP and SP with different vaules of the number of antennas M. Channel estimation using IBP gives better SINR performance over SP for M goes from 64 to 1024. At $M = 512$, IBP has 1 to 2 dB advantage over SP, depending on the number of users in the cell. As M increases, the SINR performance difference reduces. Fig. 5 shows the BER performance between IBP and SP, which is consistent with the SINR performance. For $L = 7$, $K = 18$, $C_u = 128$, there is no advantage of using IBP over SP. Fig.

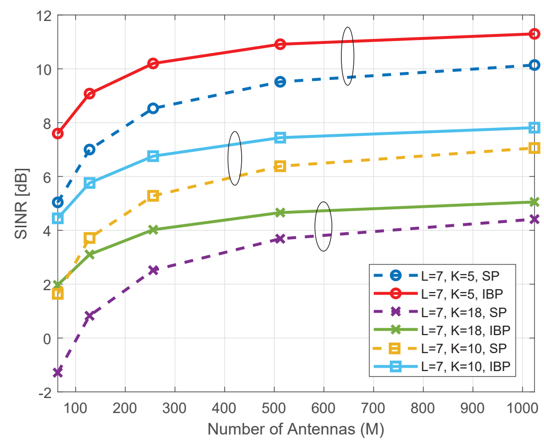

Fig. 4. SINR Comparsion between SP and IBP with Different M

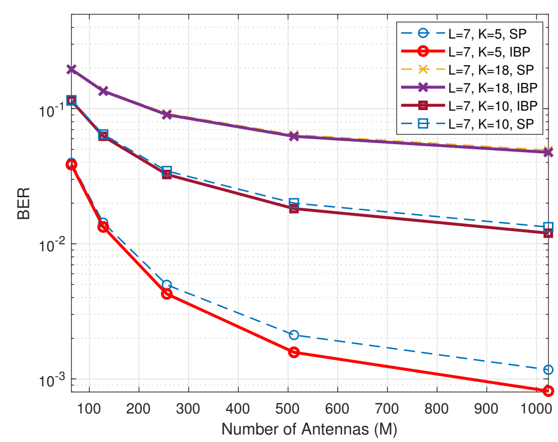

Fig. 5. BER Comparsion between SP and IBP with Different M

6 shows the SINR comparison between channel estimation using IBP and SP with different total number of users $L \times K$ in the system. Channel estimation using IBP gives about 2dB advantage for SINR performance, which are not obvious in the corresponding BER comparison for large total number of users in the system $L \times K$, as shown in Fig. 7.

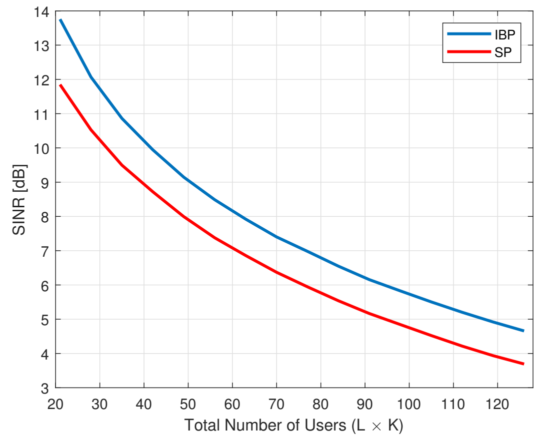

Fig. 6. SINR Comparsion between SP and IBP with Different $L \times K$

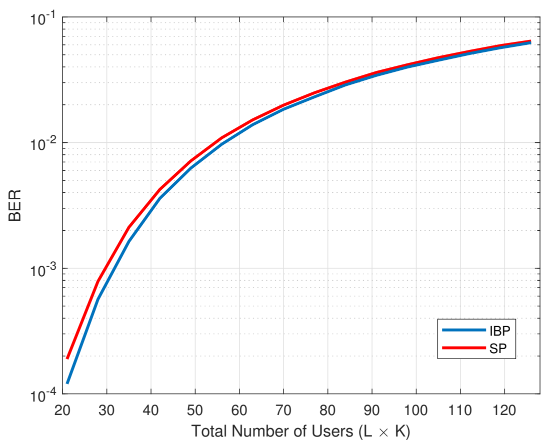

Fig. 7. BER Comparsion between SP and IBP with Different $L \times K$

VII. Conclusion

A channel estimation approach using in-band pilots for uplink DFT-Precoded massive MIMO OFDM system has been proposed and evaluated. We have compared the performance with that of using conventional time domain superimposed pilots. It was analytically shown that IBP outperforms SP in terms of MSE for channel estimation, depending on data block length, large-scale path-loss coefficient, the allocation proportion of the pilot and data power. It was also found that IBP gives slightly better BER and SINR performance over SP, depending on the number of antennas and total number of users in the system. Moreover, The removal of a small portion (e.g. $< 2\%$ of the block length) of the pilot frequencies has a negligible effect on the PAPR performance of the DFT-precoded OFDM signal. For future works, it is desirable to study the feasibility of extending the frequency domain iterative decision-directed technique proposed in [19] for SISO systems to remove the data frequency interference at the pilot locations for uplink massive MIMO systems.

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